



## **THE STUDENTS' LEARNING TRAJECTORY IN GEOMETRY CONCEPT BY USING LOCAL INSTRUCTION THEORY BASED ON REALISTIC MATHEMATICS EDUCATION APPROACH**

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### **Abstract**

This study aims to find out how learning trajectory is right for students in understanding the concept of geometry. Learning the area of triangles and quadrilateral areas through trivial problems may be easy for students, but not in the context of complex problems. Therefore, through learning with Realistic Mathematics Education (RME) approach is known learning trajectory that can help students learn geometry. From the results of the study showed that through the provision of problems that are situational and close to

the experience of students give the impact of desire and motivation and challenge students to solve problems. From this experience, students realize or do not start building material concepts that should be obtained in learning mathematics. This is a suggestion that the teaching and learning process should present problems that can take advantage of the student's previous experience before it becomes a new formal concept.

**Keywords:** *Students Learning Trajectory, Geometry Concept, Instruction Local Theory*

## A. Introduction

Mathematics is a role knowledge in building human intuition to survive and compete with their live. Through the concept of mathematics, human can build and develop innovative technologies that can facilitate their daily activities. Therefore, mathematics becomes a subject that must be studied early on and with an awareness where the focus of quantity and quality as an effort in integrating intellectuals and environments suitable for an achievement of goals (Wang & Lin, 2008). The suitability of instructional mathematics must synergize between teachers and students. Because it builds abstract mathematical concepts by teachers in the minds of students, it is not easy, especially against subjects that are just building cognitive abilities of early abstraction. Learning mathematics can be adjusted to his condition and interests such as playing video games. Playing video games does not always adversely affect students, sometimes through games students can build mathematical concepts such as geometric shapes, devise problem solving strategies for winnings, and calculate simple mathematical models (Barreto, 2017). In addition to video games, interest in learning mathematics can also be built through real activities in the environment, because through integration with students' daily activities, they can develop their personal conceptualism (Hedegaard, 1998). This means that presenting activities that exist in students' daily lives can help students build social concepts into concepts that should be understood.

The provision of opportunities for students to build concepts through supporting conditions is the principle of the process of implementing mathematics learning. For example, in low-grade learning, the process is emphasized on developing an understanding of the physical condition of the environment around students to build mathematical insights. To create this process, an adequate and conducive learning environment is needed for students so that they can build their understanding (Heuvel-Panhuizen, 2005). In addition, learning activities can also lead students to events that can build concepts from informal to formal math concepts (Freudenthal, 2002).

In the mathematics curriculum in Indonesia, the introduction of geometry concepts has been started since elementary education. The purpose of mathematics learning according to curriculum 2013 (Kemendikbud, 2013) not only emphasizes cognitive ability but more directs learning to meaningful learning through the process of observing, asking, trying, reasoning, interpreting, and creating problems. Meanwhile, according to international standards (NCTM, 2000), it is explained that there are five basic skills that students must have after learning mathematics, namely problem solving, reasoning and proof, communication, connection and representation. This standard shows that the intelligence of mathematics is not only limited to the delivery and solving trivial problems, but must have the ability and skills to achieve those standards.

These standardized skills can be realized if the paradigm of learning in Indonesian schools also changes. It means teaching by explaining the theory and then followed by examples and exercises, must be varied by learning with an innovative approach (Soedjadi, 2000). Because the teaching and learning process in this way will limit the opportunity of students in building personal concepts because it places students as objects of passive learning. As in the introduction of the concept of geometry, especially the area of triangles and quadrilaterals is not the first time known to middle school students, of course the development of problem-solving ability of triangles and quadrilateral is not difficult material. However, the study of the application of innovative and developed learning strategies in Indonesia, especially in Aceh, shows that students are still skilled in learning mathematics, especially in the direction of changes in cognitive abilities (Sari, Nasriadi, & Salmina, 2018). For many high school students as well as teachers, geometry can be difficult to learn without experience allowing them to build their own understanding. Approaches to geometry instruction, content integration, multiple representations, real-world examples, reading and writing, communication and collaboration as well as technology are ways in which students can develop their own understanding of geometric ideas (Bossé & Adu-Gyamfi, 2011).

Many learning media that can be utilized in learning such as books, props, learning videos and so on. However, learning media will not support the improvement of students' ability to master learning objectives if they are not supported by relevant activities. In learning mathematics there needs to be activities that support the construction of mathematical concepts because through activities are expected to help students develop reasoning skills at various cognitive levels by creating their own or adopting problems from different resources (Johar, Sri, & Saminan, 2015). Based on the above exposure,

mathematics learning in the classroom should be emphasized on the relationship between mathematical concepts with real experience and contextual situations close to student life. Students are organized in circumstances where their existing mathematical concepts can help shape problem solving models. With social interaction learning, it can support students to find new information that will have a positive influence on their understanding. A learning approach that can organize the situation as an integral teaching and learning process is to adopt realistic mathematical approach theory.

The realistic mathematical approach evolved from Freudenthal thinking which saw that mathematics was a human activity (Treffers, 1993). The main principle of this approach is that students should be given the opportunity to reinvent the mathematical concept as it was when it was discovered. This principle of rediscovering provides students with the opportunity to discover math concepts themselves through contextual problem submission. Through this problem students build their own models of situations to develop models for solving problems to achieve more formal mathematical knowledge (Gravemeijer, Educational Development and Developmental Research in Mathematics Education, 1994). Thus, realistic mathematics is a math teaching and learning approach that utilizes students' knowledge as a bridge to understand formal math concepts.

The realistic mathematical approach adheres to three basic principles (Gravemeijer, Real, Meaningful Mathematics, 2016). The first principle is related to guided reinvention, which is the principle by which students are given the opportunity to experience the same process as the process by which mathematical concepts are found. The second principle is related to didactical phenomenology, which is a situation where the topic of applying mathematics is given to investigate alternative actions that must be anticipated during the teaching and learning process and to see the influence that can lead to the development of the mathematical process. The third principle is related to self-development models, namely the opportunity to bridge the divide between informal knowledge of students and formal mathematics concepts.

Geometry learning is based on the thought of realistic mathematics approach, in this case facilitating students to discover the concept of geometry by utilizing their knowledge. The student's idea was explored through activities in learning involving concrete objects and an atmosphere of social interaction of learning. Therefore, it is necessary to develop a local theory that contains the learning trajectory of students in learning to understand geometry material, namely building a simple space in fourth grader elementary school students. Thus, the main focus in this research is to develop

local theories that contain the student's learning trajectory or so-called Local Instruction Theory (LIT) in developing triangular broad problem-solving skills and how students perceive learning using Realistic Mathematics Education (RME) approach.

## **B. Research Method**

This study involved 97 students in grade 7 from three different schools in Banda Aceh, Indonesia. These students have heterogeneous abilities from medium-level schools. All respondents had the same opportunity to participate in the teaching and learning process, test the results of learning, get questionnaires and interviews in the classroom conducted for three weeks.

This research procedure is carried out using design research method. According to Gravemeijer and Cobb, design research is a research method that aims to develop theories about the learning process and design steps to help the teaching and learning process (Gravemeijer & Cobb, *Design Research from a Learning Design Perspective*, 2006). The design research procedure is carried out through three stages, namely preparation (preparing for the experiment), trials of HLT (the teaching experiment), and retrospective analysis (the retrospective analysis).

The preparation stage according to the aim to design a focused and detailed Local Instruction Theory (LIT), the trial stage is to conduct HLT trials and to improve lit conductors that have been designed at the preparation stage, as well as to see the effectiveness of HLT in learning, Retrospective analysis stage is the stage of analysis of all data obtained during the research conducted in the trial in the classroom, the main purpose of this analysis is to look again at the suitability of HLT to learning and as a reference in answering research questions (Gravemeijer & Cobb, *Design Research from a Learning Design Perspective*, 2006).

In design research cycle occurs repeatedly where each cycle through the three stages described above. This research cycle will stop if the research objectives are achieved and the research answers are obtained accurately. Here is a picture of the design research cycle presented by Gravemeijer and Cobb (Gravemeijer & Cobb, *Design Research from a Learning Design Perspective*, 2006).

Qualitative data obtained in this study was collected to answer research questions about the increasing ability to solve problems of triangle and quadrilateral area seen from the influence before and after learners using realistic mathematical approaches. A total of five problems based on the area of triangles and quadrilaterals were distributed to 97

students. The distributed problem had previously been validated by three experts in the field of mathematics education consisting of two lecturers and one teacher. This is done to ensure that the problem is in accordance with the purpose of the research, namely to measure the improvement of problem-solving ability of triangle and square area.

### **C. Result and Discussion**

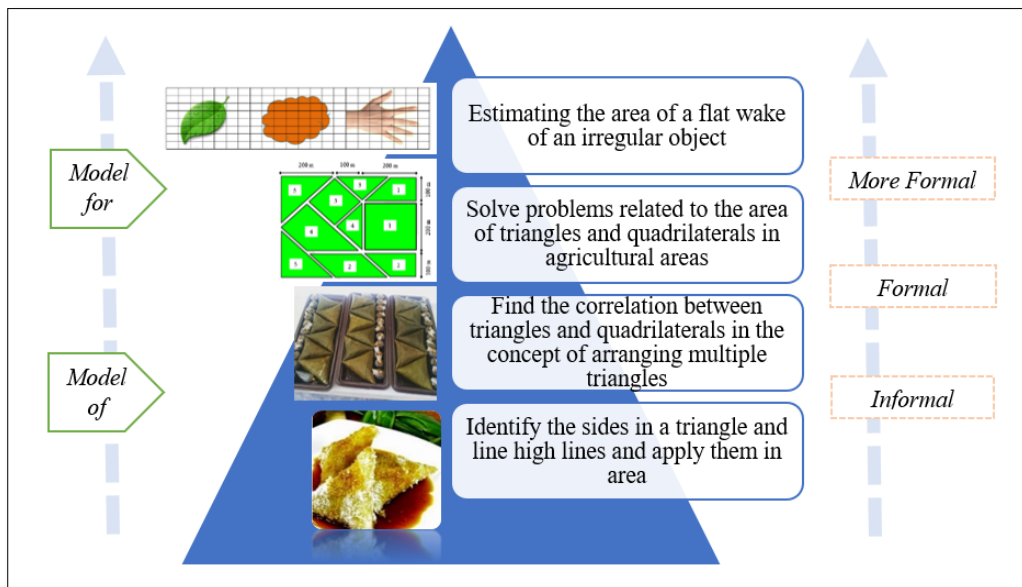
The results of this research will be discussed in three parts, namely; (1) Learning trajectory of students in studying the area of triangles and quadrilaterals, (2) quantitative reviews on improving students' ability to solve broad problems of triangles and quadrilaterals, (3) differences in student response in the teaching and learning process of students using realistic mathematical approaches.

#### *3.1. Learning trajectory in learning triangle and square area*

Based on the purpose of the research is to find the learning trajectory of students in developing problem-solving ability of triangle and quadrilateral area, then the following will be explained the results of research based on design research method in three stages.

The first stage in the research using design research method is making preparations. This stage of preparation according to Gravemeijer and Cobb aim to design a focused and detailed Local Instruction Theory (LIT) hypothesis. Before designing the LIT hypothesis was carried out preparation of LIT hypothesis consisting of; conduct literature studies, study the applicable curriculum as a draft reference, and conduct field observations related to student conditions and school atmosphere related to the feasibility of conducting research. One of the competencies that become goals in mathematics learning in the curriculum is solving problems related to the area of triangles and quadrilaterals. Based on this study, lit hypothesis was further designed in accordance with the condition of students and the atmosphere of schools in three different junior high schools in Banda Aceh.

Furthermore, LIT hypnotic and learning devices are made as a reference in field trials. Here's a simple scheme of hypnotic learning trajectory in developing students' problem-solving skills in broad triangles and square area.



**Figure 1.** the trajectory of student learning in developing problem solving skills of triangles and quadrilaterals ranging from informal knowledge to formal knowledge

The chart above shows the trajectory of student learning in developing problem solving skills of triangles and quadrilaterals ranging from informal knowledge to formal knowledge, from models of flat wake to models for the concept of wake. Furthermore, the learning trajectory hypothesis that has been designed is prepared for the field trial stage.

At the trial stage of learning trajectory hypothesis was conducted in three different junior high schools with the same way of learning to get answers whether the hypothesis can develop the ability to solve problems of students on broad problems triangles and rectangular using realistic mathematical approaches. At the first meeting, students were given a triangular model in the form of Lupis traditional cake, a traditional Indonesian cake. From this model, students are asked to identify the mathematical concepts contained in the given object. Students can identify that the model is a model of an equilateral triangle, where each side in the triangle is the same size.

Furthermore, at the second meeting, students were given the problem of arranging Lupis traditional cake to be an attractive package and have economic value. Students were asked to estimate what size and number of Lupis traditional cake could be arranged in the available cake boxes. In this case, students were asked to measure the length and width of the rectangle in advance to estimate the amount of Lupis traditional

cake that could be arranged. Here's what students answer to the problem. After a group discussion, the students then identified the types of triangles based on the length of the sides. Students formulate that in search of a triangle area, the formula used remains the same, but the determination of the height line is not the same as that of different types of triangles. Based on the traditional cake model given, students concluded that the high line is a line that is perpendicular to the side of the base.

From the problems presented in the second problem, students can estimate the number of lupis that can be arranged according to the cake box available. In addition to the lessons in baking, students found a relationship between the area of a triangle and a quadrilateral, students can also find a relationship between the area of the triangle and the quadrilateral. Here's an excerpt of the student conversation regarding the comparison of the area of triangles and quadrilaterals.

In the third meeting, students were faced with the problem of rice fields owned by five different family heads divided into triangles and squares. In such cases, students are asked to determine which head of family has the most rice harvest. In this problem, students must apply a triangular and rectangular broad formula in determining the correct answer. Based on the proposed problem, this problem is a problem that must be solved formally, namely using a triangular and rectangular area formula. The target of analysis of students' answers in this problem is the ability of students to determine the length of the sides needed in applying a broad formula. This indicates that students can already apply the broad formula of triangles and quadrilaterals in problem solving. Changes in students' abilities are explained in quantitative analysis.

The last meeting was conducted with a new problem where students were asked to estimate the area of irregular flat builds. In this meeting, students have built the concept that a flat wake is a two-inch wake measured in two measuring parameters. Thus, for flat objects can also be determined the extent. Examples of flat objects such as leaves, palms, and any object.

### 3.2. Analysis of students' self-reflection on learning

Of the 97 participants, men made up 47% and women made up 53% with an average age of 13-14 years. Based on the teaching and learning process presented with a realistic mathematical approach, students are asked to reflect on the teaching and learning process that has been presented through questionnaires and interviews. The results of self-reflection are presented in the following table.

**Table 1.** Students' experience of teaching and learning process with RME

Questions	Yes	No	No Answer
Have ever been teaching and learning with Realistic Mathematics Approach	36,0%	54,6,5%	9,27%
Feeling Challenged by Realistic Mathematical Approaches	78,3%	12,3%	9,27%

There were 88 students who responded where 69 of them or about 71.1% responded by feeling that they had gone through learning with realistic mathematical approaches, and 76 people felt challenged by realistic math learning. From the results of further interviews to respondents who feel challenged in the teaching and learning process with realistic mathematical approaches, it is known that the problems presented are quite interesting and related to the problems in their environment, so that the solution is not only relying on memorization of formulas but also requires analysis and reasoning to be able to understand the problem flow before solving it. Meanwhile, students who feel challenged by this learning answer that they are familiar with learning using lecture methods, so they expect a material explanation and examples of problems related to the area of triangles and quadrilaterals before being faced with the real problem.

**Table 2.** Students' Feelings felt after going through the Teaching and Learning Process with PMR

Question	Very Happy	Happy	Less Happy	Not Happy	No Answer
Students' feelings after going through the teaching and learning process with a Realistic Mathematics approach	6,18%	54,6%	29,8%	0%	9,27%

On questions related to students' feelings through the teaching and learning process with realistic math approach, 54.6% of students were happy with the teaching and learning process with realistic math approach and 29.8% felt ordinary. In further interviews it is known that the reason students feel happy is because they realize that the concept of mathematics can solve the problems of everyday life. Meanwhile, students who feel that learning with realistic mathematical approaches cannot rely on mathematical formulas so they feel that mathematical formulas cannot be used when solving problems proposed in learning with realistic mathematical approaches.

**Table 3.** Students' Assessment of PMR

Questions	Very Hard	Hard	Less Hard	Not Hard	No Answer
Students' assessment of learning with the Realistic Mathematics approach experienced	8,24%	42,2%	5,15%	34%	10,3%
Students' assessment of the problem presented with a realistic mathematical approach	3,09%	54,6%	5,15%	27,8%	9,27

Learning with realistic mathematical approach according to student assessment is a difficult learning shortness for students. Through further interviews it is known that students hope in learning mathematics follow the learning with a conventional approach where the teacher gives a presentation in advance about the concept or formula invited at the meeting along with examples of problems. This indicates that students are not used to building personal concepts through the submission of problems that exist in the student's daily life. By not getting used to students with the opportunity to build their own formal concepts through everyday problems or informal problems of students, learning with realistic math approaches is considered quite difficult by students.

Of the three components of retrospective analysis and students' self-reflection towards learning with realistic math approaches it is known that, students are not familiar with learning-based submission of real problems with realistic mathematical approaches. Students only ever study in groups with student worksheets but begin with learning with a conventional approach. But students feel happy and challenged enough with learning with realistic math approaches and realize that math concepts can be built from real problems and mathematical concepts should be used to solve everyday problems. Therefore, most students are quite happy with realistic mathematics learning because it turns out that learning mathematics does not rely solely on memorizing formulas and a sense of interest in mathematics but rather by applying problems that ad with mathematical concepts.

### 3.3. Qualitative review of the ability to solve triangle and square problems

There is comparison analysis before and after learning with realistic mathematical approach and effect analysis for their students' assessment by using statistical review.

Based on field studies conducted through the provision of preliminary tests and final tests it was found that the average ability of students to solve broad problems of

triangles and quadrilaterals before being taught with learning with realistic mathematical approaches was 21.75. Meanwhile, students' ability to solve broad triangular and quadrilateral problems after learning with realistic mathematical approaches was 51.66. Thus, it is obtained  $\bar{d} = 29.90$  with  $s_d = 11.96$ . Thus, obtained  $t_{\text{test}} = 24,6$ . Based on the calculation by applying the predetermined formula, it is obtained that  $t_{\text{test}} > t_{\text{score}}$  or  $24.6 > 1.66$ . Furthermore, it can be concluded that the ability of students to solve broad problems of triangles and quadrilaterals by using learning based on realistic mathematical approaches is better.

This conclusion directs that the learning trajectory hypothesis in learning designed using realistic mathematical approaches can further improve students' comprehension and problem-solving skills in broad triangular and quadrilateral problems. Aside from the differences in students' ability before and after learning the area of triangles and quadrilaterals by using learning trajectory designed can be known reliability to the problem-solving ability of students. By using the product moment correlation coefficient approach.

By using this approach, tabulation of test results before and after learning data with learning trajectory designed using realistic mathematical approach is obtained  $x_i = 2901.25$  ;  $y_i = 100514.9375$  ;  $x_i^2 = 274598.4375$ , dan  $y_i^2 = 63324$ , and it got for  $r_{xy} = 0.586$ . This  $r_{xy}$  represents the value of the correlation coefficient approach between before and after learning with the designed learning trajectory hypothesis. The  $r_{xy} > 0$  , indicates that the learning trajectory hypothesis of students is reliabel and can be a local instruction theory in developing problem-solving skills of broad triangles and quadrilateral students.

#### **D. Conclusions**

Based on the results of data analysis, it is known that there are 4 steps of Local Instruction Theory in teaching triangular and quadrilateral concepts to students aged 14-15 years. The four LIT Steps are designed by utilizing situations and conditions close to the student experience in the surrounding environment. Learning by implementing the designed LIT using the RME approach. Based on student re-selection results, rme approach is often not realized by students in their application. According to the students, although the problem presented is not easy but the student feels challenged in solving the problem because he feels the problem presented gives important meaning. From the results of the interview, students feel challenged because the problems presented

provide opportunities for students about several alternative solutions, so that the opportunity for students to utilize their mathematical models can be a bridge to the formal concept.

Qualitatively, there is an influence between the use of LIT and RME approach to student learning achievement. This is known based on test results showing that there is a fairly good positive influence on understanding between students learning and the concept of direct learning using LIT developed with the RME approach. This shows that mathematics learning designed with a real situation approach turns out to be enough to build motivation and challenges for students to know a concept in depth. So that by realizing it or not, students can find broad concepts through learning experiences or previously recognized situations. This provides advice that math learning should take advantage of situations that students recognize so that learning objectives can be conveyed properly.

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